“Pi” in Your Face

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Central High School, Victoria, VA

Grade: 10

Time Allotment: One 90-minute block

Overview:
Students having difficulty understanding, applying, and memorizing circular formulas involving pi will be given direct instruction and hands-on activities for remediation.

Subject: Geometry

Lesson Objectives:

- Students will define pi as the ratio of the circle’s circumference and its diameter.
- Students will define and locate the circle’s diameter, radius, and circumference.
- Students will measure in centimeters.
- Students will recognize and use pi and approximation symbols.
- Students will determine and state the approximate decimal value for geometric pi.
- Given a diameter measure and a compass, students will construct circles.
- Students will use rules of equations to transform formulas.

Standards:

This lesson addresses the Standards of Learning for the State of Virginia for Geometry. These SOLs can be found at http://www.pen.k12.va.us

- G.8 The student will
  a) investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
  b) prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
  c) use properties of quadrilaterals to solve practical problems.

- G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

- G.10 The student will investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and the area of a sector, and may be drawn from applications of architecture, art, and construction.

- G.12 The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.

Media Components:

- *Math Factor: General Quadratic Relation: Circles and Ellipses* "Defining Circles with the General Term", United Streaming – http://www.unitedstreaming.com

- *Project Mathematics: The Story of Pi* "Computation of Pi", United Streaming – http://www.unitedstreaming.com

Materials:

Per Student Group for Introductory Activity:

- 1 compass
- 1 ruler with centimeter measurement
- vocabulary word strips (attached)
- pencil
- scotch tape
- calculator
- ink pen
- worksheet – one per student (attached)
- 1 aluminum pie pan (any size)

Per Student Group for Learning Activity
(Many of the same materials can be used for both activities)

- ruler
- compass
- pen and pencil
- calculator
- worksheet – same sheet as above (attached)
- 1 ½ yards of S'getti String Plastic Lacing
- old magazine or newspaper for work surface
- scissors
- 1 set of colored pencils (blue, green, and red)
- chart paper or blackboard to show formulas

Prep for Teachers:

- Provide compasses that have centimeter markings. Students should already have a working knowledge of construction of circles using the compass.

- The S'getti strings Plastic Lacing works best for the Learning Activities hands-on activity. This type of plastic string can be found in any craft store.
- Preview the video and cue to the correct spot.
- When using media, provide students with a **Focus for Media Interaction**, a specific task to complete and/or information to identify during or after viewing of video segments, Web sites or other multimedia elements.

Introductory Activity:

1. As a check for understanding of vocabulary, do the following activity. Group the students into pairs. Provide for each pair an aluminum pie pan (any size), ruler, worksheet (attached), pencil, calculator, and vocabulary word strips (attached), and scotch tape.
2. **Say:** "Today we are going to open the 'pi' cabinet. As any good 'pi' baker knows the secret is in the crust and the crust is a pan full of knowledge. Let's review the parts of a 'pi' pan. We are going to watch a short video clip. The narrator is going to define the term *circle*." Provide a **Focus for Media Interaction** by saying, "Listen for the definition and be ready to give me that definition". **START** *Math Factor: General Quadratic Relation: Circles and Ellipses:* "Defining Circles with the General Term" at the beginning and **STOP** at 0:26 when the narrator says "…a fixed distance from the center".
3. **Ask:** "What definition did the narrator give for the term *circle*?" (*a collection of a set of points that are fixed distance from the center*) **Say:** "In a previous lesson we discussed the term that defined the distance from the center of the circle to the outside edge. What was that term?"(*radius*) Draw the circle and radius example on the board. "What term did we use when we drew a line completely through the center from one side of the circle to the other?" (*diameter*). "Finally, we discussed a term that defined the distance all the way around the circle. What was this term?" (*Circumference*) "Great! We will come back to the video in a few minutes to discover some relationships between these parts of the circle."
4. **Say:** "Okay now let's put on our aprons and bake up some 'pi'. Take your pie pan and turn it over so the bottom is facing up. You and your partner take your vocabulary strips. Draw using your ruler and pencil one diameter and one radius on the bottom of your pie pan. Then use your vocabulary strips to label these lines and also label the circumference. When finished labeling, use your ruler to measure the diameter of the bottom of the pie pan to the nearest ½ centimeter. Record this measurement correctly on your worksheet. I'll give you a few minutes to complete the task."

Learning Activities:

1. **Ask:** "What are the two main parts of a pie?" (*crust and filling*) "In geometry there is a special 'pi' that also has two main parts. This pie is symbolized with the Greek letter that looks like this (*Draw pi symbol on the board*.) and is spelled 'pi' (*also write on the board*). Make note of this on your worksheet."

2. **Ask:** "Based on what we have labeled on the pie pans, what do you think the two main parts of this special 'pi' are?" (*Circumference, diameter*) **Ask**: "Why didn't we choose radius as one of the main parts?" (*lead students to the understanding that the radius is part of the diameter*)
3. **Say:** "Just like in baking a pie, the crust has to go in a certain place and the filling has to go in a certain place. Otherwise you will have a gooey mess. In order for the geometry pi not to be a gooey mess, its parts must be in a certain place or order."
4. **Say:** "There is a relationship, called a ratio, between pi's parts- the circumference and the diameter. Remember ratio is just a fancy word meaning making a comparison. We will use the fraction line to show this comparison. Let's write this ratio on your worksheet." (*draw on board pi symbol* = $\frac{\text{Circumference}}{\text{Diameter}}$)

5. "Now let's think about this special pi a little bit more. The value of pi is so fundamental people have been trying to figure out its exact numerical value accurately. Watch this next clip and tell me the first person who tried to figure out the numerical value of pi." **START** *Project Mathematics: The Story of Pi:* **"**Computation of Pi*"* at the beginning and **PAUSE** at 0:16 when the narrator says "…Archimedes". **Ask:** "Who was the first person to attempt to find out the value of pi?" (*Archimedes*) **Say:** "From this next segment of video, be able to tell me how he found the value of pi." **RESUME** video and **STOP** at 1:24 when the narrator says "…a fairly good approximation for pi". **Ask:** "How did Archimedes find the value of pi?" (*He continued to use regular polygons inside and outside a circle. He kept doubling the amount of sides until he had a polygon with 96 sides. He found an estimate for pi correct to two decimal places. The fraction 22/7 is still used today*.) **Say:** "Wow! I'm not sure everyone heard all that information. Let's rewind that segment and listen to it again and see if someone can explain how Archimedes came up with the approximate value for pi." **REWIND** to 0:16. **REPLAY** and **STOP** at 1:24. "Can someone explain how Archimedes found the approximate value? Well, this seems like a lengthy way to discover the value."

6. **Say:** "Now we are going to complete an activity while keeping the same partner. You should have the following materials at your table: compass, ruler, your worksheet, 1 foot of cotton string, colored pencils including red, green, and blue, and an old magazine. Use your old magazine as a press and turn your worksheet over on top of it so the clean white side is showing. We will now draw three circular pies to help us discover the mathematical value of the geometric pi."
7. **Say:** "Begin by drawing a small dot near the center of your paper. This dot will be all of your pies' origins or centers, which mean all of your pies will look like concentric circles."
8. **Say:** "We are going to make our first circle with a diameter of 14 centimeters." **Ask:** "What should we set our compass on?"(*7 cm*) If students struggle with this answer remind them that the compass will need to be set on the radius value which is ½ of the diameter. **Say:** "Place the point of the compass on the dot you created on your paper then construct your circle. Take your green colored pencil and trace this circle. This circle will represent an apple pie."

9. **Say:** "Our second circle, which will represent a blueberry pie will have a diameter of 7 cm." **Ask**: "What should we set our compass on?" (*3.5 cm)* Remind students of the importance of being accurate with compass usage. "Remember to place the point of the compass on the same dot to create your second circle. Trace this circle with the blue colored pencil."
10. **Say:** "Finally let's create a cherry pie that is smaller in circumference than your blueberry pie." **Ask:** "What other main part of the pie will also change and be smaller in size?" (*diameter*) **Ask:** "So what could your compass setting be for this pie?"(*accept any answers greater than 0 and less than 3.5 cm)* "Use this setting to create your circle. Remember to use the same center as your other circles. Trace this smallest circle with a red colored pencil. This circle represents the cherry pie."
11. **Say:** "Now, we have constructed all of these pie circles to remind you of our key vocabulary, as well as to sneak in some compass construction practice."
12. **Say:** "Look at just the apple pie…your green circle… the big circle. Make a guess as to what you think the circumference is. Remember the circumference is the perimeter of the circle. Now take your plastic lace and work with your partner to lay the lacing on the circumference of that circle. <u>You must be precise in this procedure</u>. Carefully, as one person holds the lacing down the other will mark with a pen where the lacing needs to be cut. Cut the lacing at this mark so that the lacing represents the measurement entirely around the circle – thus the circumference. Measure the amount of lacing in centimeters."
13. **Say:** "Record this measurement and the diameter measure for this pie on the front of your worksheet. Repeat the plastic lacing procedure on the blueberry and cherry pies. Each time, record the circumference and diameter on the worksheet."
14. **Say:** "Pile all materials to the side except for the worksheet, pencil and calculator. We are going to transform your "gooey" measurements into a geometric "pi". Look at the middle of your worksheet where you wrote the pi ratio." pi symbol = $\frac{\text{Circumference}}{\text{diameter}}$

    Refer to the board where the teacher wrote the same information. "Remember the fraction line means to divide. Let's substitute in the measurement values of the circumference and diameter from our apple pie into this ratio." (*students should write 44/14 – 44 represents the circumference and is the numerator or top number of the ratio and the 14 represents the diameter and is the denominator or bottom number of the ratio*.)
15. **Say:** "Use your calculator to determine the value of this pi. Remember it's the top value divided by the bottom. Write this value on your worksheet. Before we discuss your discovery repeat this procedure with your other two pies."
16. **Say:** "Do you see any patterns on your paper with your discovery? Turn to the groups around you and see what they discovered." (*all answers should be close to 3.14*) **Say:** "Why didn't all of our "pi's" come out the same? Didn't we use the same recipe, same formula?" (*discuss variables that may have influenced the procedures and results*)
17. **Say:** "Going back over our recipe, our formula, we discovered that the two main parts of our geometric "pi" are in a ratio. The circumference of a circle when

divided by its diameter comes out to approximately 3.14. Don't forget this means we must use the approximation symbol when using values in this formula." (*Draw the approximation symbol on the board and refer students to it on their worksheet*.)

The following steps are designed for advanced students or for a transition lesson.

18. **Say:** "We are almost ready to eat, but let's take this one step further and add some meringue to our 'pi's'. Given what we know, we should be able to use the approximate decimal value of pi and a diameter measurement of a circle to discover its circumference." **Ask:** "How can I use what I know about the rules of equations to have the circumference value alone on one side of the equation?" (*multiply both sides of the equation by "d"*) **Say:** "Let's do that with our pi formula."
19. **Say:** "Our new formula is diameter multiplied by pi = the circumference. This formula can be found at the bottom of your worksheet. So if the 'pi's' that we bake in geometry have the approximate value of 3.14 we can use this new formula to discover circumference. Since we already know the approximate diameter of the aluminum pie pans we used to review our vocabulary, let's use that same measure and our new discovery of the approximate decimal value of pi to determine the pie pan's circumference. Fill in the blanks and circles on your worksheets."

Culminating Activity:

1. Teacher should instruct students to move the pile of materials back to their workspace.
2. **Say:** "Now let's put some '*Pi' in My Face!* Create an original pie and put that knowledge in my face. Show me exactly how you figured out the value of geometric pi using your materials i.e. string, compass, ruler, etc. but you must also show me through the use of the formula.
3. **Say:** "Raise your hand when you are ready to sling some pie my way."

Note: For reinforcement, students could receive an individual pie or a slice of pie to eat.

Cross-Curricular Extensions:

Writing:
Students will explain the process of determining pi in the form of a written report.

Science:
Have students measure the circumference of a tree, and calculate its diameter. Ask them if there is a way to approximate the age of the tree, diameter or radius of the tree.

History:
What other Greek letters have been used in math or science? Research the history of pi.

Technology:
Students will develop a basic program that will determine the circumference of a circle when given its diameter.

Community Connections:

- Invite a mathematician from a local college to discuss the concept of pi.
- Students could conduct an on-line poll to find out how many people really understand what pi is.
- Invite someone from the computer industry into the class to discuss how pi is used in the manufacture of large computers. They could discuss the history of computers and/or their careers.
- URL's to explore:

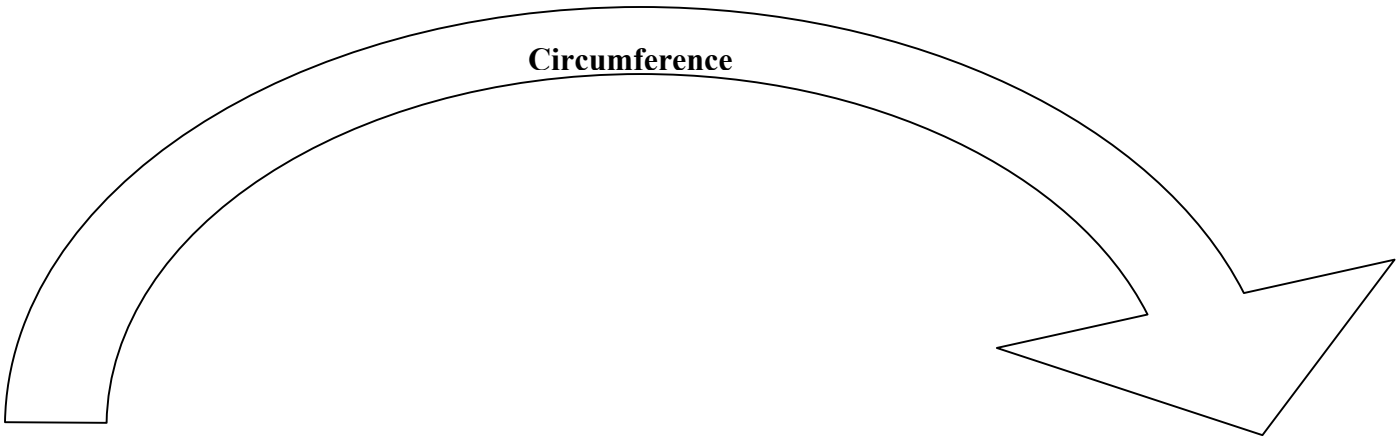
1. http://mathforum.org/isaac/problems/pi1.html - This site is from the Math Forum collection. Math Forum's goal is to build an online community of teachers, students, researchers, parents, educators, and citizens at all levels that have an interest in math and math education.
2. http://www.figurethis.org/challenges/c15/challenge.htm - In this activity students use measurements of the girth, or circumference, of a tree to see which tree is large enough that a car could be driven through it. This crazy but fun idea allows students to visualize and work with the areas and circumferences of circles.
3. http://mathcentral.uregina.ca - An Internet service for mathematics teachers and students from Kindergarten to grade twelve. It is a meeting place for teachers to share resources, a service to teachers, students and parents who may need an answer to a mathematical question.
4. http://www.unidata.ucar.edu/staff/blynds/rnbw.html - This site is a tutorial developed by Beverly Lynds, co-PI for Project Skymath of the University Corporation for Atmospheric Research. The images and charts that are provided help to clarify the description, and really bring the words to life.
5. http://www.coolmath.com - This commercial site includes sections for both younger and older students, games, calculators, lessons, and more. Other sections for teachers and parents include tips and discussion forums.

- Other United streaming videos that are related to the subject:
    1. *Project Mathematics: The Story of Pi* - United Streaming
    2. *Videomath: Circles* – United Streaming
    3. *Mathematical Eye: Circles* – United Streaming

Vocabulary Word Strips

Radii

Diameter



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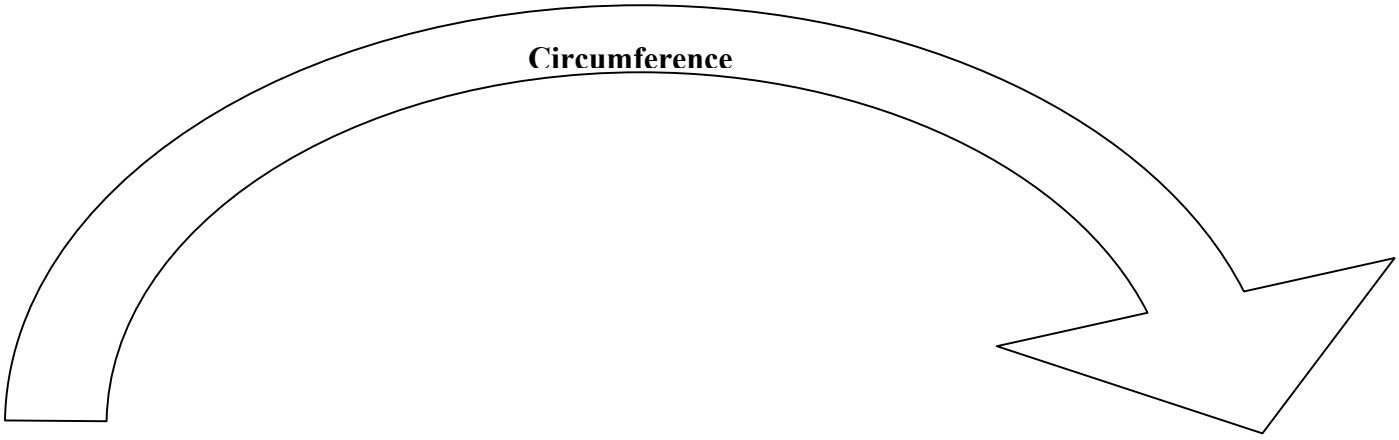


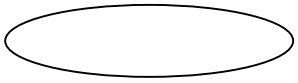
Worksheet for "Pi" in Your Face

- Pie Pan Measurements: d = _____ cm

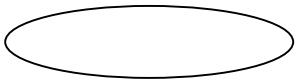
- Greek letter/symbol _____ spelled _____

  Π = _____

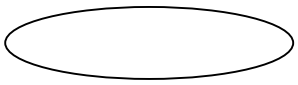
- Apple Pie C = _____ cm; d = _____ cm

  Π = C/d = _____ ≈ 

- Blueberry Pie C = _____ cm; d = _____ cm

  Π = C/d = _____ ≈ 

- Cherry Pie C = _____ cm; d = _____ cm

  Π = C/d = _____ ≈ 

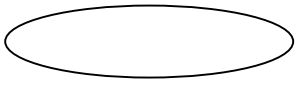
- Bonus Activity:

Pie Pan Π ≈ 3.14; d = _____cm; C = _____ cm

d (Π) = (c/d)d

dΠ = C

_____ . 3.14 = C

 = C