

# It's A Peeling

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**Overview:** In this lesson students will discover the formula for the surface area of a sphere through hands-on exploration. Prerequisite: This lesson should be preceded by the study of surface area of prisms, pyramids, cylinders and cones. The student must also be able to measure the diameter and radius of a circle and apply the formula for area of a circle. ( $A = \pi r^2$ ). Students must also be able to recognize that the great circle of a sphere occurs when the sphere is cut into two congruent parts, i.e. the sphere is cut in half.

**Grade Level:** 9-12

**Time Allotment:**  
One 90-minute block

**Subject Matter:**  
Geometry

## Learning Objectives:

The student will be able to:

- Visualize and model the surface area of a sphere
- Use estimation strategies to approximate the surface area of a sphere
- Derive the formula for the surface area of a sphere

## Standards:

Virginia Standards of Learning

<http://www.pen.k12.va.us/go/Sols/math.html#Geometry> G.13: The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems.

## Media Components:

ITV Program: *Math Vantage #11*: Volume and Surface Area

## Web Sites:

Euclid's Elements, Using the Geometry Applet:  
<http://aleph0.clarku.edu/~djoyce/java/elements/usingApplet.html>

This site allows students to drag the endpoints ( $A$  and  $B$ ) of the diameter of the sphere with length  $AB$  to and visualize how that changes the size of the sphere.

## Materials and Teacher Preparation:

For each group of 4 students:

- 1 inch piece of removable scotch magic tape
- Knife to cut orange
- Scissors
- 2 pieces of centimeter graph paper, sample attached
- Plain paper (11"x17" does well)
- Orange (as spherical as possible) (Navel oranges are easiest to use.)
- Plastic wrap or baggy
- Wet paper towels or sponge for cleanup

## Prep for Teachers:

Prior to teaching, bookmark the Web site and in order to take advantage of this applet, be sure that you have enabled Java on your browser. If you disable Java, or if your browser is not Java-capable, then the illustrations will still appear, but as plain images. You will not be able to drag the images and watch as they change.

Make a transparency of the attached Goode Homolosine map.

To demonstrate various applications of surface area, have a variety of balls (baseball, basketball, soccer ball, etc.) on hand to use as models.

Next cue the videotape, Math Vantage #11 to the section on Surface Area. This comes right after you the short section on volume and the Math Vantage Logo. Practice pausing the video after each figure (rectangular prism, triangular prism, square pyramid, cylinder, and cone) because this is a very short segment and it goes by quickly.

When using media, provide students with a **FOCUS FOR MEDIA INTERACTION**, a specific task to complete and/or information to identify during or after the video segments, Web sites, or other multimedia elements.

### **Introductory Activity:Setting the Stage**

#### **Step 1**

Review the concept of surface area as the sum of the areas of the two-dimensional shapes that form the sides of the figure. To reinforce this concept tell students they are going to view a short video clip about surface area.

#### **Step 2**

1. Provide students with a FOCUS FOR MEDIA INTERACTION by saying: “We will now watch a video clip to help us review the concept of surface area. As you watch the first segment I want you to notice how to find the surface of three-dimensional figures.”
2. PLAY the video with SOUND OFF and PAUSE the video when you see the rectangular prism. SAY: “How do we find the surface area of a rectangular prism?” Student responses should include finding the area of the three different rectangles and multiplying by two, or finding the area of each rectangular shape and adding them up.
3. RESUME the video with SOUND OFF, view the triangular prism and PAUSE the video. SAY: “How do we find the surface area of a triangular prism?” Student responses should include adding the areas of the four rectangles to the areas of the two triangles.
4. RESUME the video with SOUND OFF and view the square pyramid then PAUSE the

- video. SAY: “How do we find the surface area of a square pyramid?” Student responses should include add the area of the square base to the area of the four triangles.
5. RESUME the video with SOUND OFF and view the cylinder. PAUSE the video. SAY: “How do we find the surface area of the cylinder?” Student responses should include add the area of the two circles from the top and bottom of the cylinder to the area of the rectangle that forms the side of the cylinder. SAY: “What shapes do you think would make up the surface area of a cone?” Answers should include a part of a circle and the circle from the base of the cone.
  6. RESUME the video with SOUND OFF and STOP the video when you see the parts of the cone. REWIND the video to the beginning of this segment (where you see the Math Vantage Logo) play with SOUND ON. You will hear “A way to measure Surface Area of some three-dimensional figures is to compute the areas of the two-dimensional shapes forming the sides of the containers. For example, the rectangular prism, the triangular prism the square pyramid, the cylinder and the cone.” STOP the video when you see the cone. SAY: “Today we are going to figure out how to find the surface area of a sphere and will see the rest of this video clip after we have tried to discover a formula for the surface area of a sphere.”

#### **STEP 3**

1. Ask students to list as many real life examples of spheres as they can in one minute. Examples might include baseball, marble, malt ball, golf ball, basketball, globe, fireball, etc. Then have students brainstorm about the various mathematical properties of spheres. Examples might include descriptions of the radius, diameter, volume, surface area, great circle, circumference, and center. Have students identify these characteristics with respect to a baseball.

Volume: the inside portion of the baseball or sphere ( $v = \frac{4}{3} \pi r^3$ )

Center: the point equidistant from all points on the surface of the sphere

Radius: the distance from the center to a point on the surface of the sphere

Diameter: the distance from one point on the sphere through the center to another point on the sphere.

Surface Area: the number of square units in the leather covering of the baseball.

Great Circle: the largest circle created by slicing through the center of the sphere

Circumference: the distance around the sphere

#### STEP 4

2. Hand each group of students an orange and several sheets of centimeter graph paper. Ask students to predict the surface area of their orange in square centimeters. Have each group of students record its prediction on the board for later discussion.
3. Examine a baseball, basketball, soccer ball, and a Goode Homolosine map with the students so they might discover several ways to cover the surface of a sphere (their orange). Tell students: you are going to have a contest to design a covering for an orange using the least amount of material but covering the orange completely. The students will use the centimeter graph paper to cover the orange. Recommend that students cut their 1 inch piece of tape into strips so that it is easily removed. Students will compete to create the covering that most accurately depicts the actual surface area. When the groups have finished covering their orange they should remove and flatten the covering and count the number of square centimeters. Agree on some ground rules about estimating partial squares. Have each group reveal the number of square centimeters used to cover their orange.
4. Record these results on the board next to their estimates. Remember, just because a group has the smallest number of square centimeters, that doesn't mean their covering is the most accurate.

## Learning Activities

### Step 1

1. Say: "We will now determine the winner of the orange contest." Hand each group of students a knife, piece of plastic wrap or baggy, and large sheet of paper. Have students slice the orange through the center to create two congruent halves. The exposed section of the orange is the great circle.
2. Place one half of the orange in the baggy or cover it with plastic wrap to keep the paper from getting soggy. Instruct students to trace the great circle on their paper 6 times.
3. Tell students that we are now going to peel the oranges and ask them to predict how many of these great circles will be filled with orange peel. Say: "Students, you may eat the orange but be sure to account for all of the orange peel as you place it in these great circles."
4. Have students fill in all the gaps with small pieces of orange peel so that the circles are completely covered. Students may remove the white pith in order to facilitate this process. Once all students have completed this activity, have them share the number of great circles covered by their pieces of peelings. (The orange peel will cover 4 great circles)
5. Say: "How can you use this information in determining the surface area of each orange?" Ask students to now compute the surface area by finding the area of the circles that are covered. (They should measure the diameter in centimeters and divide by two to obtain the radius of the great circle. Then use the formula for area of a circle =  $\pi r^2$  and multiply it by 4.) Have students record their calculated surface area. Determine which group's estimate was the most accurate.
6. Say: "Do you think that if we can find the area of the great circle of any sphere, we can multiply it by four and find the surface area of that sphere?" (yes)

## Step 2

1. Say: “We will now look at an interactive applet that shows the sphere getting larger as you increase its diameter.” Open the Euclid web site and enlarge the applet by typing u while your cursor is over top of the image. Maximize the applet and drag the objects by the red points. Allow students to ask questions as you demonstrate the changes made by dragging the object.
2. Next, ask students if they can visualize the great circle by using these diagrams. Drag the sphere over top of the semicircle and show them that the diameter of the great circle and sphere are the same no matter how you change the size of the sphere or diameter.
3. Discuss what happens to the surface area of the sphere as the diameter changes? (as one increases and decreases so does the other.)
4. Say: “Can we determine the formula for the surface area of a sphere from this demonstration? How do we know that our calculated surface area of each orange is correct? Even though all students should have four great circles covered with orange peel, does this guarantee that our formula is correct?”

## Culminating Activity

Students will now watch the final segment of the Math Vantage #11 video and determine if their formula for area of a sphere is correct. Provide students with a FOCUS FOR MEDIA INTERACTION by saying, “We will now watch the rest of the video clip which is about the surface area of a sphere. First, we will watch the video without the sound to see if our predictions are correct then we will watch it again and listen for the description about surface area of a sphere.” PLAY with SOUND OFF the last segment of the video. PAUSE when you see the hostess with a fishing pole. SAY: “Do you think that our prediction (that it takes four great circles to cover a sphere) about the surface area of a sphere is correct? Let’s watch this segment again and listen for the explanation.” REWIND and PLAY with

SOUND ON. STOP the video after you hear the hostess say “the area of those four circles will completely cover the surface of a sphere.”

## Cross-Curricular Extensions

**Technology:** Have students create a spreadsheet with three columns titled radius, area of the great circle and surface area of a sphere. Insert the formulas for each column and fill down so that each column is calculated based on the radius. (See the last page for a sample chart.) This can also be done in the lists of a graphing calculator.

**Language Arts:** Students can create limericks about spheres or other three-dimensional objects.

**Music:** Have students create a song about spheres.

**Art:** Have student practice drawing spheres and other three-dimensional objects.

**Social Studies:** Students can learn about the use of spherical objects throughout history. For example, when did spherical mini balls become replaced with non-spherical bullets? Why did this occur?

**Science:** Students can study spherical objects that occur in nature, such as dandelion “puff balls”, walnuts, and other spherical objects.

**Architecture:** Students can research Buckminster Fuller and his work with geodesic domes.

## Community Connections

1. If there is a candy company in your area, a representative could come in and talk about the shape of his or her product. What are the advantages and disadvantages to spherical shaped candies?
2. Students can go to the local stores and try to find as many spherical objects as possible. Have students make a list of all of the objects they find and report back to class with their discoveries.

3. Contact a manufacturer of balls and discover how they are created. What is the surface area of the ball made of? Why is that material chosen?

### STUDENT MATERIALS

*per team*

- 1 inch piece of removable scotch magic tape
- Knife to cut orange
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- Plain paper (11"x17" does well)
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Navel oranges are easiest to use.
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| Radius | Area of Great Circle<br>( $3.14 \times \text{Radius}^2$ ) | Surface Area of Sphere (4*Area of Great Circle) |
|--------|---|---|
| 1      | 3.14  | 12.56   |
| 2      | 12.56   | 50.24   |
| 3      | 28.26   | 113.04  |
| 4      | 50.24   | 200.96  |
| 5      | 78.5  | 314   |

QuickTime™ and a  
GIF decompressor  
are needed to see this picture.